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MODIFIED MAXIMUM ENTROPY SPECTRAL ANALYSIS OF BANDLIMITED SIGNALS IN LOWPASS NOISE

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The mean properties of the modified maximum entropy method (MEM) spectral estimator incorporating variable delay are developed for the case of a bandlimited (BL) complex signal in lowpass noise. Asymptotic cases of long filter length and degeneration to the white noise case are examined in detail. The modified MEM is shown to reproduce the input spectrum (to within a scaling factor) for $\Delta = 1$ and to provide a spectral whitening function as Δ grows very large. Additionally, examples of spectral distortion for $\Delta > 1$ are shown and qualitatively explained.

delta greater than unity

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delta

CONTENTS

INTRODUCTION

This paper examines a class of spectral estimates denoted as the modified maximum entropy method (MEM) for inputs consisting of bandlimited (finite nonzero bandwidth) signals in lowpass additive noise. This class of inputs is one of substantial practical application. For instance, a signal considered to be a Gaussian process of some bandwidth may be transmitted through a channel exhibiting the "1/F" type frequency absorption. In another example, a pure sinusoid might be sent through a media whose time-varying properties serve to modulate the sinusoid and broaden its frequency characteristics.

The spectral estimate termed in this paper as "modified" MEM differs in two ways from the original MEM as developed by Burg.[1] First, the modified MEM incorporates a variable delay Δ, whereas the method developed by Burg is for unity delay. The motivation for removing the constraint of unity delay is as follows. As shown by Van den Bos,[2] MEM spectral analysis is equivalent to the all-pole linear prediction spectrum for unit delay. This MEM spectrum is often derived using the coefficients of a linear prediction filter which are optimal in the minimum mean-square error (MMSE) sense. Often, however, there are situations for which this MMSE filter is constructed using a delay value which is substantially greater than unity. An example is given by Alexander and Satorius[3] whereby the bandlimited signal characteristics of a spectrum are separated from the lowpass noise properties on the basis of an MMSE prediction filter with variable delay. Since the MEM spectral estimate is easily derivable from the filter coefficients, we examine the properties of the maximum entropy spectrum for delays other than $\Delta = 1$. Since MEM is properly defined only for unity delay, the method incorporating a variable delay will be denoted as modified MEM.

A second difference between MEM and modified MEM is a power scaling term. As developed by Burg, MEM contains a numerator scaling factor which is the MMSE power due to the optimal linear prediction filter. The inclusion of this scaling factor causes the MEM to be a true power spectral estimator because the integration of the MEM spectrum over all frequencies produces the true input process power. The modified MEM spectrum does not contain this scaling term and, as pointed out by Griffiths,[4] is not a true power spectral estimate. However, it is shown further in Reference 4 that for $\Delta = 1$ the modified MEM spectrum is related to the MEM spectrum through a constant scale factor, namely, the MMSE of the optimal linear prediction filter for the input process. Thus, many general properties (such as spectral shape, peak frequency locations, order estimation, etc. . . .) of MEM spectra may be examined equivalently through the modified MEM for unity delay.

The work presented in this paper is concerned with two areas: an examination of the mean properties for the unit delay modified MEM for the case of a bandlimited signal in a lowpass noise background and the effects of variable delay length upon the modified MEM spectrum for this input process.

ANALYSIS

LINEAR PREDICTION FILTER IMPULSE RESPONSE

In this section we develop the structure of the variable-delay modified MEM spectrum through analysis of the associated MMSE filter properties. Thus, we develop first the expression for the linear prediction filter impulse response and then illustrate how this quantity determines the resultant modified MEM spectrum.

Consider first the classical Wiener MMSE prediction problem: given a sequence of data values extending L-samples into the past, $\{x(k), x(k-1), \ldots, x(k-L+1)\}$, we wish to form the best estimate (in the MMSE sense) of the sequence value Δ-samples into the future, $\hat{x}(k+\Delta)$, using a linear discrete filter. It is well-known that these optimal filter coefficients, $h(k)$, may be obtained by solving the discrete Wiener matrix equation

$$R\underline{h} = \underline{P}_\Delta \quad . \tag{1}$$

In Equation 1, $\underline{h}$ is the weight vector containing the optimal coefficients, R is the $L \times L$ autocorrelation matrix of the input process and $\underline{P}_\Delta$ is the L-length vector containing the autocorrelation elements $\phi_{xx}(\ell + \Delta)$, where $\ell = 0, 1, \ldots, L-1$. Equivalently, Equation 1 may be expanded into L simultaneous equations of the form

$$\sum_{k=0}^{L-1} \phi_{xx}(\ell - k)\, h(k) = \phi_{xx}(\ell + \Delta) \quad , \; \ell = 0, 1, \ldots, L-1. \tag{2}$$

A method for analytically solving Equation 2 for the $h(k)$ used in several recent works[3,5,6] is the method of undetermined coefficients. Specifically, this method was used by Alexander and Satorius[3] in their solution for the MMSE filter impulse response for an input consisting of a complex bandlimited signal in lowpass noise. The solutions obtained in Reference 3 were complex weight vectors, corresponding to the complex input signal. However, Zeidler, Satorius, et al,[6] showed that for widely separated, complex exponential frequencies the weight vector solutions for real sinusoids could be obtained by a superposition of the composite complex exponential weight solutions. A similar assumption is invoked here: that for complex bandlimited signals widely separated we can obtain weight solutions for real bandlimited signals by a similar superposition. Explicit examination of the bandlimited interference mechanisms will be done in a future paper. The present work is limited to the case of a single bandlimited complex signal in lowpass noise.

The autocorrelation function for this specific input is given by

$$\phi_{xx}(\ell) = \sigma_N^2 e^{-\alpha_N |\ell|} + \sigma_S^2 e^{-\alpha_S |\ell|} e^{j\omega_S \ell} \tag{3}$$

where

σ_N^2 = noise mean-square power

σ_S^2 = bandlimited signal mean-square power

α_N, α_S = correlation parameters of noise and signal, respectively

ω_S = radian frequency (relative to sampling frequency) of complex bandlimited signal.

Using the inversion integral, the spectrum $S_{xx}(z)$ corresponding to $\phi_{xx}(\ell)$ in Equation 3 is given by[3]

$$S_{xx}(z) = \frac{2\sigma_N^2 e^{-\alpha_N} \sinh \alpha_N}{(z - e^{-\alpha_N})(z^{-1} - e^{-\alpha_N})} + \frac{2\sigma_S^2 e^{-\alpha_S} \sinh \alpha_S}{(z - e^{-\alpha_S + j\omega_S})(z^{-1} - e^{-\alpha_S - j\omega_S})} \tag{4}$$

This form of the input power spectrum will be important in future applications.

Reference 3 has derived the general solution for the MMSE filter coefficients $h_\Delta(k)$ for an input process given by the autocorrelation function of Equation 3. The Δ-notation in $h_\Delta(k)$ signifies the Δ-dependance of the impusle response through $B_1(\Delta)$ and $C_1(\Delta)$ which are themselves functions of Δ. The solution for $h_\Delta(k)$ is given by

$$h_\Delta(k) = B_1(\Delta)\, z_1^k + B_2(\Delta)\, z_2^k + C_1(\Delta)\, \delta(k) + C_2(\Delta)\, \delta(k - L + 1)$$

$$k = 0, 1, \ldots, L - 1 \tag{5}$$

where $\{z_1, z_2\} = \{e^{-\mu + j\theta_1}, e^{\mu + j\theta_1}\}$ are the zeroes of the input spectrum and $\{B_1, B_2, C_1, C_2\}$ are given as the solution set *to a 4 X 4 matrix equation. By examining the asymptotic* case of long filter length L, the solution given by Equation 5 simplifies and gives a physical insight into the resulting filter structure.

For long filter lengths,* the solution for the weight vector reduces to[3]:

$$h_\Delta(k) = B_1(\Delta)\, e^{-\mu k}\, e^{j\theta_1 k} + C_1(\Delta)\, \delta(k) \tag{6}$$

where, in Equation 6,

$$B_1(\Delta) = \left\{ \frac{\left[1 - e^{\alpha_N - \mu} e^{j\theta_1}\right]\left[1 - e^{\alpha_S - \mu} e^{j(\theta_1 - \omega_S)}\right]}{e^{\alpha_N - \mu} e^{j\theta} - e^{\alpha_S - \mu} e^{j(\theta_1 - \omega_S)}} \right\}$$

$$\left[e^{-\alpha_N \Delta} - e^{(-\alpha_S + j\omega_S)\Delta}\right] \tag{7a}$$

$$C_1(\Delta) = e^{-\alpha_N \Delta} - \left\{ \frac{\left[1 - e^{\alpha_S - \mu} e^{j(\theta_1 - \omega_S)}\right]\left[e^{-\alpha_N \Delta} - e^{(-\alpha_S + j\omega_S)\Delta}\right]}{e^{\alpha_N - \mu} e^{j\theta_1} - e^{\alpha_S - \mu} e^{j(\theta_1 - \omega_S)}} \right\} \tag{7b}$$

Once the impulse response $h_\Delta(k)$ is derived, the transfer function $H_\Delta(z)$ is obtained simply by the z-transform operation:

*Long filter length signifies $L >> 4/\mu$, where μ is given by the location of the input spectral zeros.

$$H_\Delta(z) = \sum_{k=0}^{L-1} h_\Delta(k) z^{-k} \quad . \tag{8}$$

Using Equation 6 and assuming a very long filter length, $H_\Delta(z)$ becomes

$$H_\Delta(z) = \frac{B_1(\Delta)}{1 - e^{-\mu + j\theta_1} z^{-1}} + C_1(\Delta) \quad . \tag{9}$$

The frequency response of the transfer function then is obtained by evaluating $H_\Delta(z)$ around the unit circle (making the substitution $z = e^{j\omega}$). The frequency response is thus dependent upon the value of Δ chosen.

MODIFIED MEM FOR VARIABLE DELAY

Analogous to the spectral estimation method defined in Reference 4, we define the variable-delay modified MEM spectrum as $Q_\Delta(z)$, where

$$Q_\Delta(z) = \left| 1 - z^{-\Delta} H_\Delta(z) \right|^{-2} \quad . \tag{10}$$

Similarly,

$$Q_\Delta(\omega) = Q_\Delta(z) \Big|_{z = e^{j\omega}} . \tag{11}$$

In Equation 10, $H_\Delta(z)$ is given as in Equation 8 and $z^{-\Delta}$ signifies a delay operator of Δ samples . For the case of a complex bandlimited signal in lowpass noise, $h_\Delta(z)$ is given by Equation 4, $H_\Delta(z)$ from Equation 8, and the resulting modified $Q_\Delta(z)$ spectrum computed from Equation 10. For the case of long filter length, the weight vector $h_\Delta(z)$ from Equation 6 is used, leading to $H_\Delta(z)$ given by Equation 9. The remainder of this paper invokes the condition of long filter length and examines analytically and graphically the resulting solutions for $Q_\Delta(z)$.

ASYMPTOTIC CASES

LONG FILTER LENGTHS

Makhoul[7] shows that in the limit as $L \rightarrow \infty$, the MEM spectrum with $\Delta = 1$ will provide an unbiased mean estimate of the true power spectrum for any arbitrary input. Thus, utilizing a delay $\Delta = 1$ and $L \rightarrow \infty$, the modified MEM reproduces the original power spectrum to within a scale factor. Figure 1 shows an input spectrum $S_{XX}(\omega)$ for a bandlimited signal in lowpass noise where the given parameters refer to the autocorrelation function of the spectrum as given in Equation 3. Figure 2 then shows the resulting modified MEM spectrum produced by three values of delay. In Figure 2a, for $\Delta = 1$ the modified MEM spectrum is a replication to within a scale factor of the input spectrum $S_{XX}(\omega)$. For the case of non-unity delay (as shown by Figures 2a and 2c) a significant spectral distortion is introduced by increasing delay. However, the spectral peak corresponding to the center frequency of the bandlimited signal is retained by the non-unity delay spectra. The spectral distortion away from the signal center frequency produced by increased delay may be qualitatively explained by the following analysis. Theoretically, we can examine the extrema of $Q_\Delta(z)$ by solving the equation

$$\frac{d}{dz} Q_\Delta(z) = 0 \tag{12}$$

for the locations of the extrema. With $Q_\Delta(z)$ given by Equation 10, this leads to the condition

$$A_\Delta(z) \frac{d}{dz} \overline{A}_\Delta(z) + \overline{A}_\Delta(z) \frac{d}{dz} A_\Delta(z) = 0 \tag{13}$$

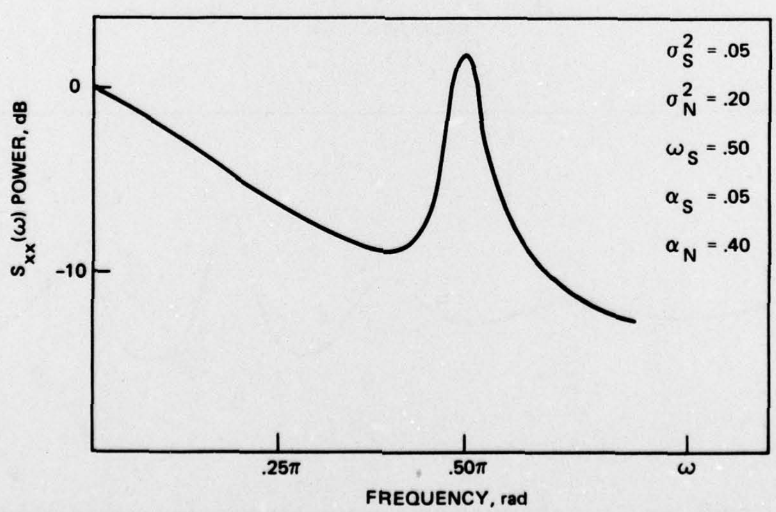


Figure 1. Input power spectrum for bandlimited complex signal in lowpass noise.

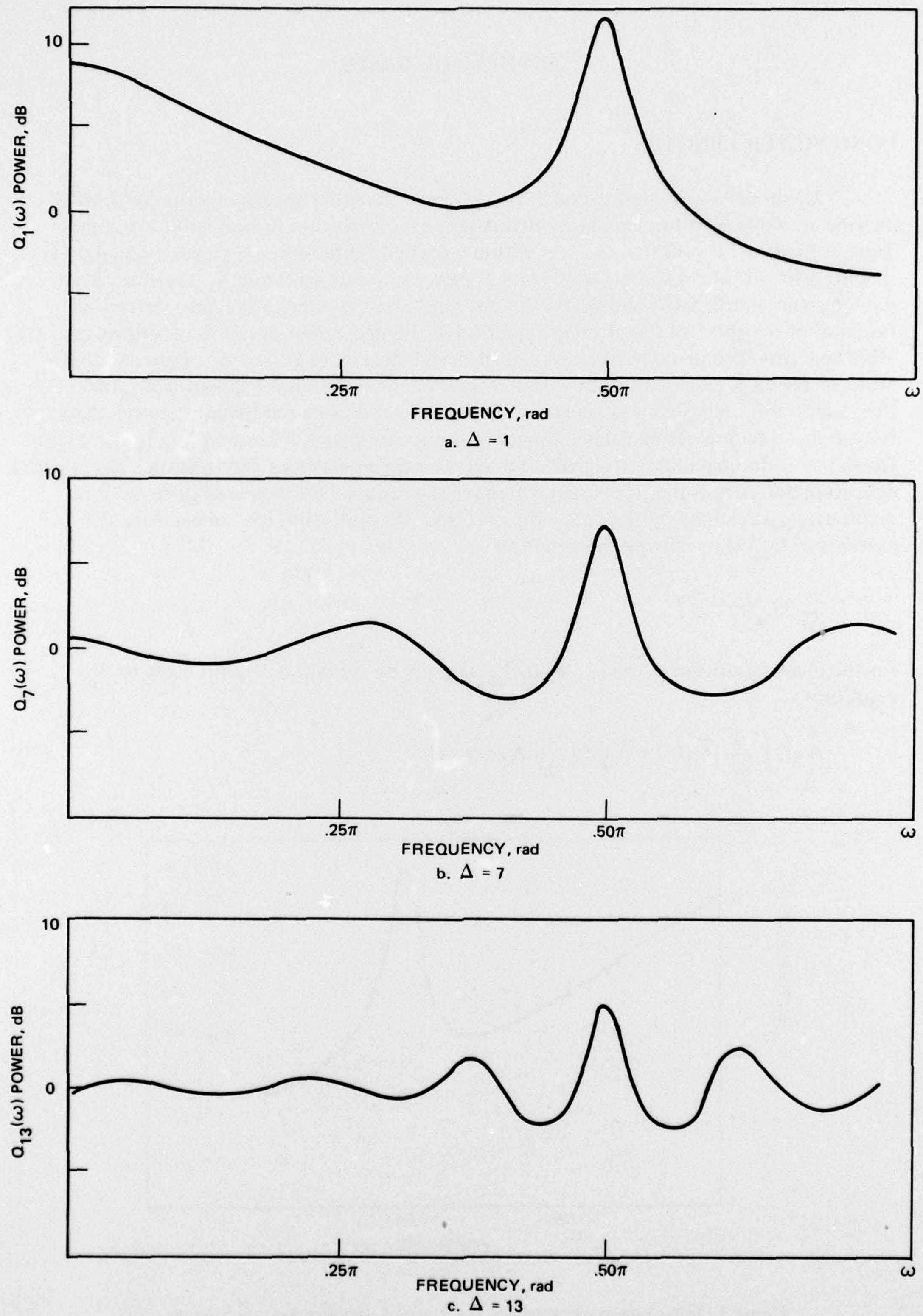


Figure 2. Modified MEM spectra for input spectrum of Figure 1.

where, in Equation 13, the overbar implies complex conjugation and $A_\Delta(z)$ is given by

$$A_\Delta(z) = 1 - z^{-\Delta} H_\Delta(z) \quad . \tag{14}$$

In general, the solution to Equation 13 is a formidable analytical task and will not be pursued further at this time. However, some general properties of the extrema of $Q_\Delta(z)$ may be examined by considering the polynomial $A_\Delta(z)$ in Equation 14. Since it is known that $Q_1(z)$ reproduces the input spectrum within a scale factor, the polynomial $A_1(z)$ must be of the proper order to estimate correctly the input spectrum. Thus any increase of delay past $\Delta = 1$ will serve only to introduce additional z-domain roots into Equation 14. These newly introduced roots thus appear in $Q_\Delta(z)$ as additional poles in the z-plane, which serve to distort the spectrum away from the signal center frequency. As Δ is increased by one, the order of $Q_\Delta(z)$ is thus increased by one, producing an extra pole at some location $z_p = e^{a+jb}$. Then as $Q_\Delta(z)$ is evaluated around the unit-circle ($z = e^{j\omega}$) to form $Q_\Delta(\omega)$, two additional extrema (one maxima and one minima) appear in the range $-\pi \leqslant \omega \leqslant \pi$. From this development for an arbitrary delay $\Delta = D$, a total of $2(D - 1)$ additional extrema appears in $Q_\Delta(\omega)$ and considerable distortion of the signal input may result.

Although the spectral distortion as shown in Figure 2 is, in general, undesirable, there are some situations in which these can be beneficial results by increasing delay beyond $\Delta = 1$. An example is shown beginning with Figure 3, which is considerably lower power bandlimited signal than that of Figure 1. In $S_{xx}(\omega)$ and in $Q_1(\omega)$ shown in Figure 4a the lowpass noise dominates the spectrum. However, by increasing Δ past unity (Figures 4b and 4c) the effect of the lowpass noise on the resulting $Q_\Delta(\omega)$ estimate may be made smaller and smaller. The power in the bandlimited signal is not attenuated greatly for increasing delay and the signal center frequency is estimated correctly by $Q_\Delta(\omega)$ for increasing delay. The

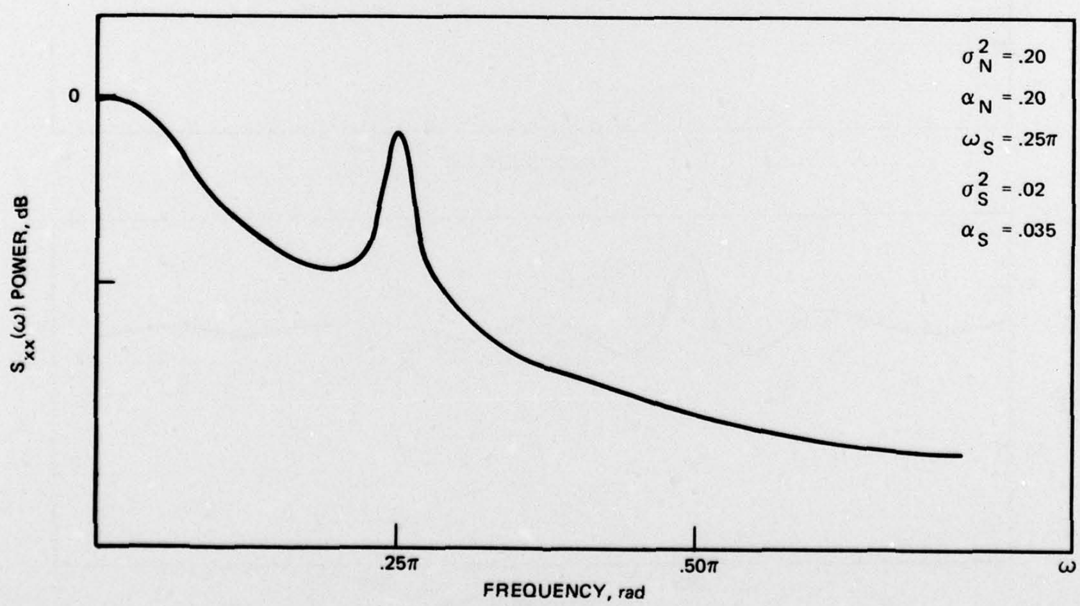


Figure 3. Input spectrum for bandlimited complex signal in lowpass noise.

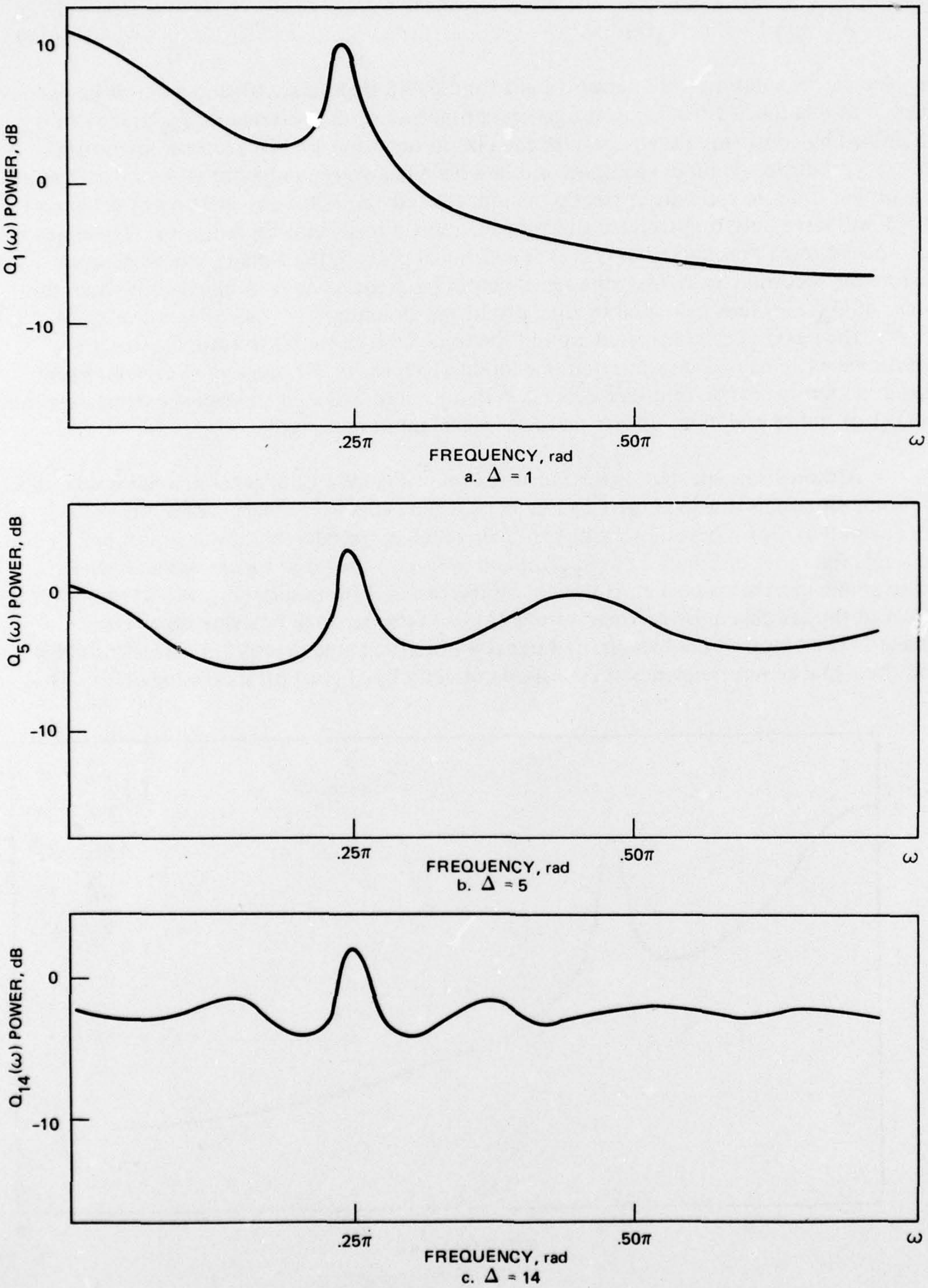


Figure 4. Modified MEM spectra for input spectrum of Figure 3.

decorrelation of the lowpass noise is due to the properties of the linear prediction MMSE filter $H_\Delta(\omega)$, as developed in Reference 2. The bandlimited signal has a longer correlation distance than the lowpass noise and thus is not decorrelated so severely. An asymptotic property of $Q_\Delta(\omega)$ for large delay lengths may also be inferred from the plots in Figures 2 and 4. As Δ is made significantly larger than the correlation distance of the bandlimited signal, the linear prediction filter $H_\Delta(\omega)$ is no longer able to extract the bandlimited signal properties from the input series, and, in the limit for large delay $H_\Delta(\omega)$, approaches zero for all ω. This latter property may be seen in the limits of $B_1(\Delta)$ and $C_1(\Delta)$ for $\Delta \to \infty$. From Equations 7a and 7b,

$$\lim_{\Delta \to \infty} B_1(\Delta) = 0 \tag{15a}$$

$$\lim_{\Delta \to \infty} C_1(\Delta) = 0 \tag{15b}$$

which gives from Equation 9,

$$\lim_{\Delta \to \infty} H_\Delta(\omega) = 0 \tag{16a}$$

and thus from Equation 10,

$$\lim_{\Delta \to \infty} Q_\Delta(\omega) = 1 \tag{16b}$$

Thus the result of using a very long delay value is to whiten perfectly the $Q_\Delta(\omega)$ spectrum, which is illustrated in Figures 2 and 4. For increasing Δ-values, the spectral ripples grow lower in amplitude and closer in frequency until, for very long delay values, the resulting spectrum is effectively white. The delay necessary to whiten the input spectrum must be chosen to be much greater than the longest correlation distance of any bandlimited signal present. For a bandlimited signal with bandwidth parameter α_S, as given by Equation 3, the delay must be given by $\Delta >> 1/\alpha_S$ to achieve this whitening.

BANDLIMITED SIGNAL IN WHITE NOISE

The analytical results for a bandlimited complex signal in white noise are easily obtained from the previous derivations through a simple limiting operation. From Equation 3 we see that the autocorrelation function for a complex bandlimited signal in white noise is obtained if one allows α_N to approach infinity. The effect on the weight vector solution is such that for long filter lengths, $C_1(\Delta)$ from Equation 7 vanishes and $h_\Delta(k)$ is given by

$$h_\Delta(k) = B_1'(\Delta)\, e^{-\mu k}\, e^{j\theta_1 k} \tag{17}$$

where

$$B_1'(\Delta) = \lim_{\alpha_N \to \infty} B_1(\Delta) = e^{(-\alpha_S + j\omega_S)\Delta}\left[1 - e^{\alpha_S - \mu}\, e^{j(\theta_1 - \omega_S)}\right]. \tag{18}$$

But as shown in Reference 3 for white noise, the exponential frequency θ_1 is exactly equal to the bandlimited signal center frequency ω_S. This gives for $h_\Delta(k)$ the expression

$$h_\Delta(k) = e^{(-\alpha_S+j\omega_S)\Delta}\,(1 - e^{\alpha_S-\mu})\,e^{-\mu k}\,e^{j\omega_S k} \;. \tag{19}$$

The transfer function $H_\Delta(z)$ for long filter length is then given by Equation 9 as

$$H_\Delta(z) = \frac{e^{(-\alpha_S+j\omega_S)\Delta}\left[1 - e^{\alpha_S-\mu}\right]}{1 - e^{-\mu+j\theta_1}\,z^{-1}} \;. \tag{20}$$

The modified MEM spectrum $Q_\Delta(z)$ is then given by (using Equations 10, 14 and 20):

$$Q_\Delta(z) = \frac{1}{A_\Delta(z)\,\bar{A}_\Delta(z)} = |\,A_\Delta(z)\,|^{-2} \tag{21}$$

where

$$A_\Delta(z) = \frac{1 - e^{-\mu+j\omega_S}\,z^{-1} - (1 - e^{\alpha_S-\mu})\,e^{(-\alpha_S+j\omega_S)\Delta}\,z^{-1}}{1 - e^{-\mu+j\omega_S}\,z^{-1}} \;. \tag{22}$$

A simple closed form expression for $Q_\Delta(z)$ may be obtained for the special case $\Delta = 1$. Thus setting $\Delta = 1$ in Equation 22 and substituting in Equation 21, we can calculate $Q_1(z)$, which, after simplification, becomes

$$Q_1(z) = e^{\alpha_S-\mu}\left[\frac{z^2 - 2ze^{j\omega_S}\cosh\mu + e^{j2\omega_S}}{z^2 - 2ze^{j\omega_S}\cosh\alpha_S + e^{j2\omega_S}}\right] \;. \tag{23}$$

Using the relationships developed thus far, we may now verify analytically the equivalence (within a scale factor) of $Q_1(z)$ and $S_{xx}(z)$ for the case of a complex bandlimited signal in white noise. The spectrum $S_{xx}(z)$ for this input may be obtained from Equation 4 by allowing $\alpha_N \to \infty$. This gives

$$S_{xx}(z) = \sigma_N^2 + \frac{2\sigma_S^2\,e^{-\alpha_S}\sinh\alpha_S}{(z - e^{-\alpha_S+j\omega_S})(z^{-1} - e^{-\alpha_S-j\omega_S})} \;. \tag{24}$$

Combining terms in Equation 23,

$$S_{xx}(z) = \sigma_N^2\,\frac{\left\{z^2 - 2ze^{j\omega_S}\left[\cosh\alpha_S + (\sigma_S^2/\sigma_N^2)\sinh\alpha_S\right] + e^{j2\omega_S}\right\}}{z^2 - 2z\cosh\alpha_S\,e^{j\omega_S} + e^{j2\omega_S}} \;.$$

The spectral zeros of $S_{xx}(z)$ thus are given by solving for the roots of

$$z^2 - 2ae^{j\omega_S}\,z + e^{j2\omega_S} = 0 \tag{25}$$

where, in Equation 25,

$$a = \cosh \alpha_S + \left(\frac{\sigma_S^2}{\sigma_N^2} \right) \sinh \alpha_S \quad . \tag{26}$$

Solving Equation 25 via the complex quadratic formula gives

$$z_1 = e^{j\omega_S} \left[a - \sqrt{a^2 - 1} \right] = e^{j\theta_1} e^{-\mu} \tag{27a}$$

$$z_2 = e^{j\omega_S} \left[a + \sqrt{a^2 - 1} \right] = e^{j\theta_1} e^{\mu} \quad . \tag{27b}$$

These equations give the relations

$$\theta_1 = \omega_S \tag{28a}$$

$$e^{-\mu} = a - \sqrt{a^2 - 1} \tag{28b}$$

$$e^{\mu} = a + \sqrt{a^2 - 1} \quad . \tag{28c}$$

The first relation states that the complex exponential component of the weight vector solution for white noise is at exactly the bandlimited center frequency, regardless of signal bandwidth. The last two relations may be used to derive the following:

$$\cosh \mu = \frac{e^{\mu} + e^{-\mu}}{2} = a \quad . \tag{29}$$

Substituting this relation into Equation 24 for $S_{xx}(z)$ we get

$$S_{xx}(z) = \sigma_N^2 \left[\frac{z^2 - 2ze^{j\omega_S} \cosh \mu + e^{j2\omega_S}}{Z^2 - 2ze^{j\omega_S} \cosh \alpha_S + e^{j2\omega_S}} \right] \quad . \tag{30}$$

Comparing this result with the expression for $Q_1(z)$ as given by Equation 23, $Q_1(z)$ and $S_{xx}(z)$ are indeed equivalent to within a scale factor:

$$Q_1(z) = \frac{e^{\alpha_S - \mu}}{\sigma_N^2} \left[S_{xx}(z) \right] \quad . \tag{31}$$

Figure 5 shows the power spectrum of a moderately bandlimited signal in white noise. Figure 6a then gives the modified MEM spectrum for $\Delta = 1$ and the reproduction of $S_{xx}(\omega)$ may be seen clearly. The parameter $e^{-\mu}$ for this case equals 0.783 and thus the scaling factor becomes

$$\frac{e^{\alpha_S - \mu}}{\sigma_N^2} = 4.33 = 6.4 \text{ dB}$$

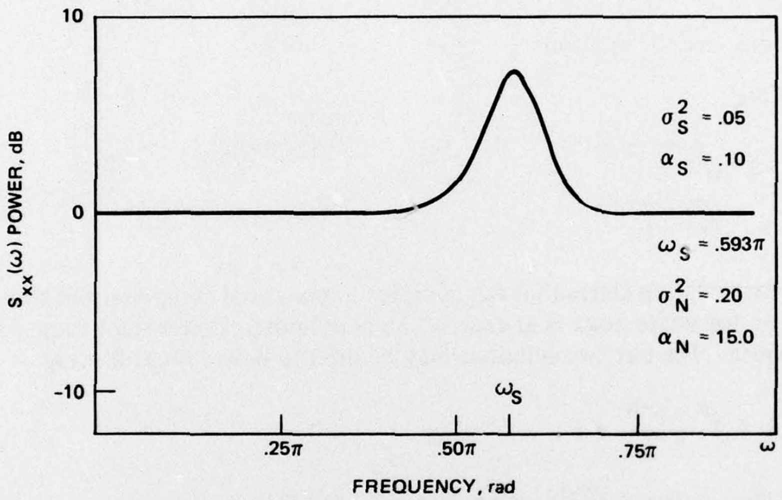


Figure 5. Spectrum of bandlimited complex signal in white noise.

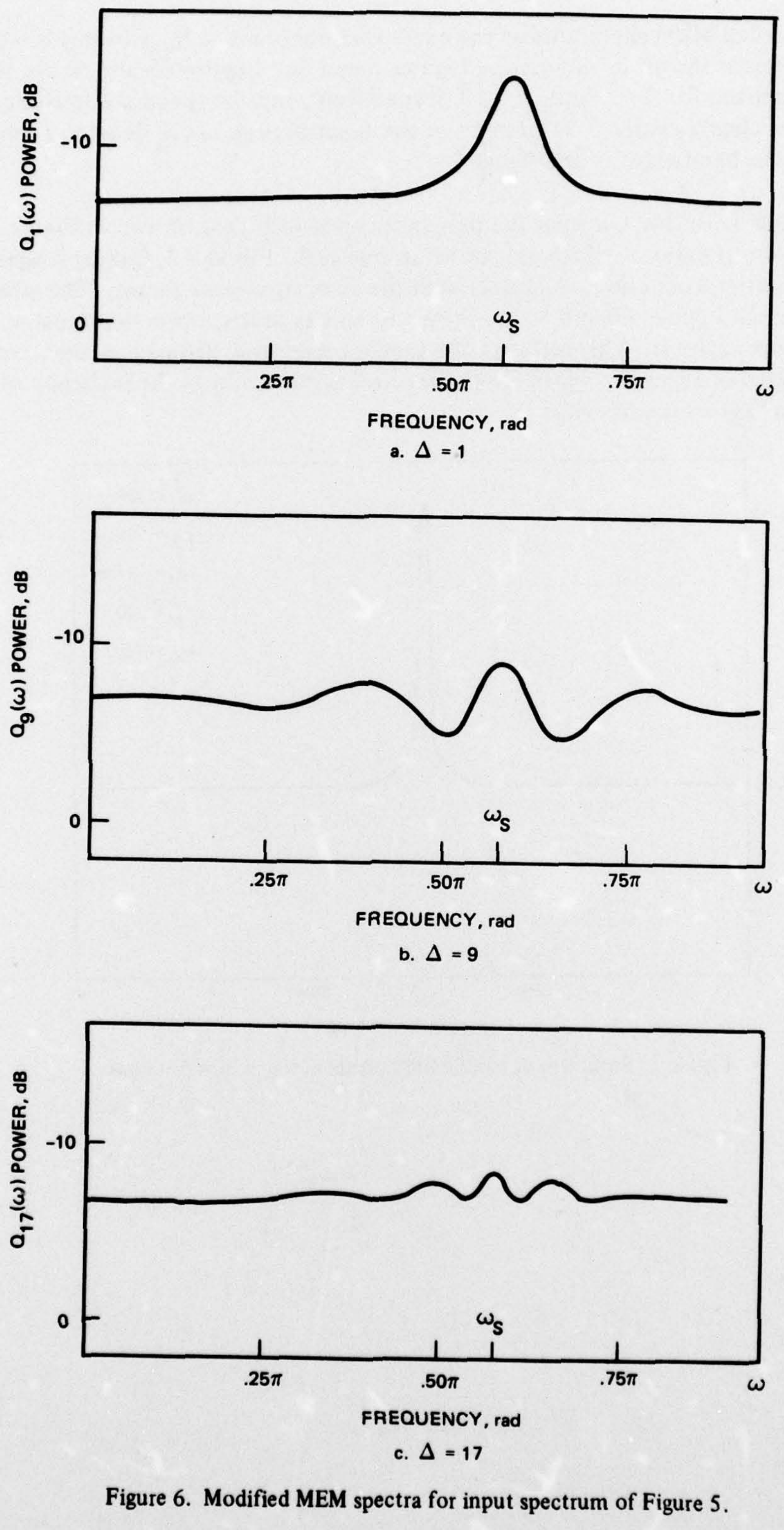


Figure 6. Modified MEM spectra for input spectrum of Figure 5.

Thus the modified MEM spectrum has the exact spectral shape as $S_{XX}(\omega)$ but is 6.4 dB higher in power, as shown by comparing Figures 5 and 6a. Figures 6b and 6c are the modified MEM spectrum for $\Delta = 9$ and $\Delta = 17$, respectively, and the spectral flattening due to longer delay is clearly evident. The height of the spectral peak at ω_S decreases rapidly with delay due to the bandwidth of the signal.

In contrast to this, consider the narrower bandwidth peak shown in Figure 7, the modified MEM estimates of which are shown in Figure 8. For $\Delta = 1$, $Q_1(\omega)$ is again seen to give an excellent reproduction of $S_{XX}(\omega)$ with the associated scale factor. The effects of increased delay in Figures 8b and 8c are shown to be less drastic upon the signal peak than in the preceding example. This is due to the longer correlation distance of the narrower bandwidth signal of Figure 7. However the spectral rippling due to the inclusion of additional poles in $Q_\Delta(\omega)$ is still evident.

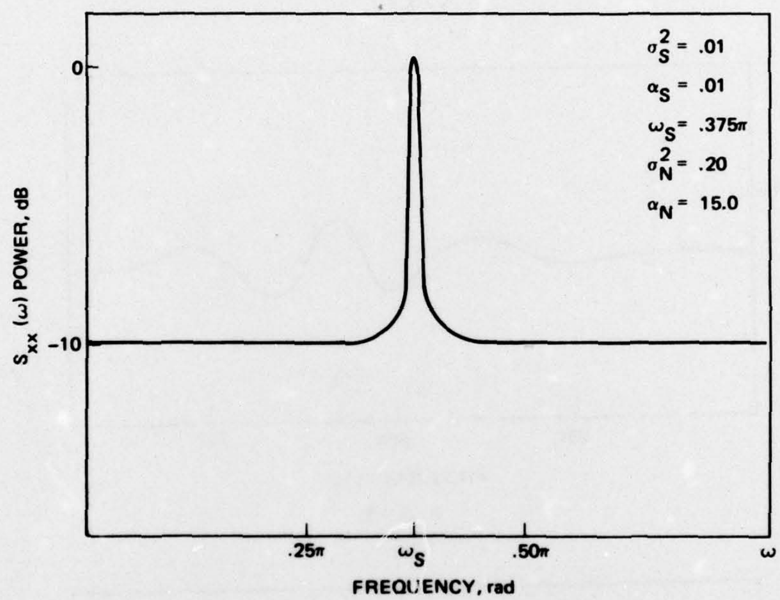
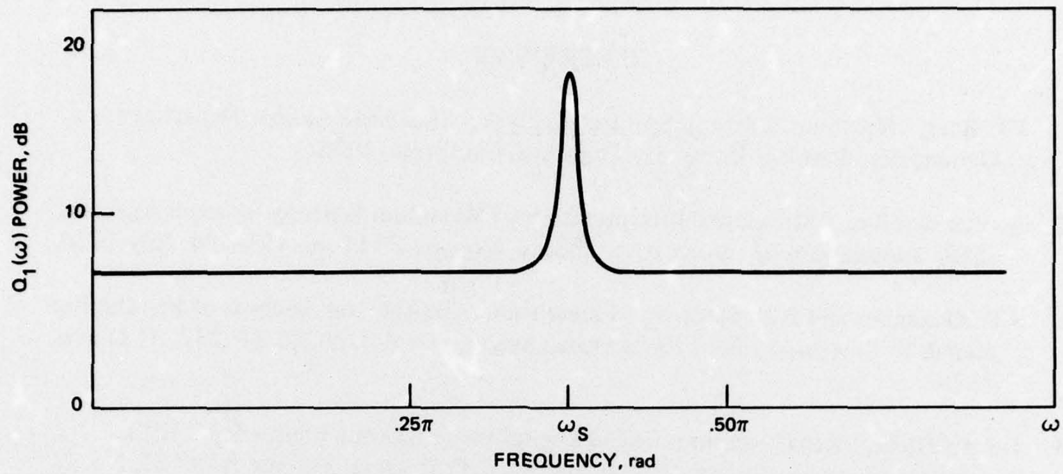
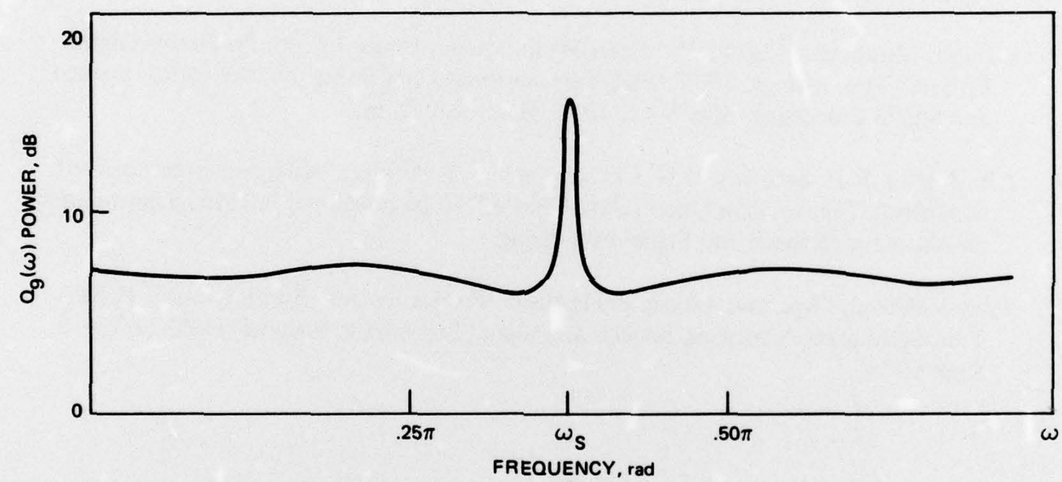


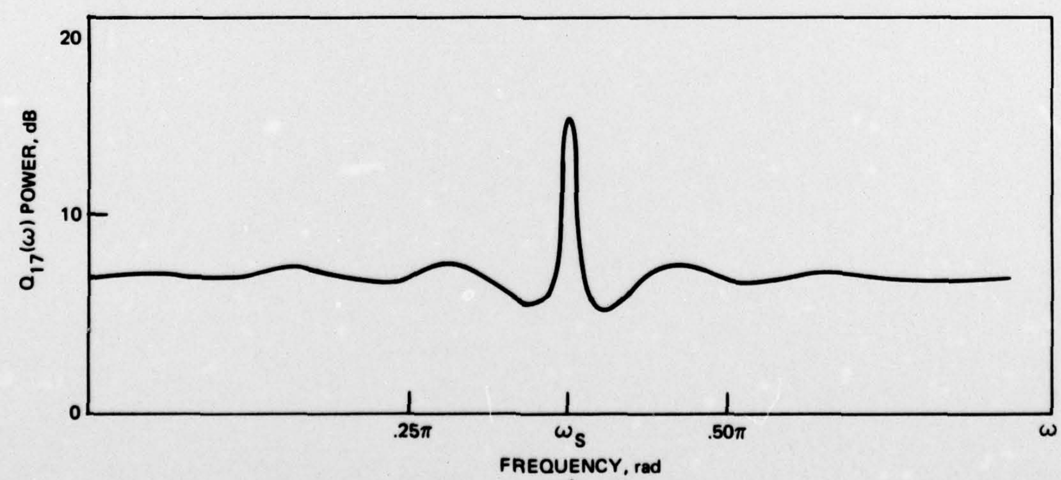
Figure 7. Spectrum of bandlimited complex signal in white noise.



a. $\Delta = 1$



b. $\Delta = 9$



c. $\Delta = 17$

Figure 8. Modified MEM spectra for input spectrum of Figure 7.

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